

# Economics 704a Lecture 10: Optimal Policy III, Liquidity Trap and Unconventional Policy I

Adam M. Guren

Boston University

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# Optimal Policy in the NK Model: Outline

1. Optimal Monetary Policy Without Commitment
  - 1.1 Welfare
  - 1.2 The “Divine Coincidence”
  - 1.3 Breaking the Divine Coincidence and the  $\pi - Y$  Tradeoff
  - 1.4 Principles of Discretionary Monetary Policy
2. Monetary Policy In Practice: 2021-22
3. Optimal Monetary Policy With Commitment
  - 3.1 Time Inconsistency and the Gains From Commitment
  - 3.2 Inflation Bias and Commitment
  - 3.3 The  $\pi - Y$  Tradeoff With Commitment: A Simple Rule
  - 3.4 The  $\pi - Y$  Tradeoff With Commitment: The General Case
  - 3.5 Policy Rules and Communication

# Gains From Commitment With Zero Target Output Gap

- Last class: General idea of time inconsistency and inflation bias if  $k > 0$ .
- However, even if  $k = 0$ , commitment can be useful.
  - By taking advantage of way that expectations affect output and inflation, can *improve output-inflation tradeoff*.
- Will tackle this in two ways:
  1. Today: Central bank commits to simple rule  $x_t^c = -\omega u_t$  which nests discretion if  $\omega = \frac{\kappa}{\kappa^2 + \vartheta(1 - \beta\rho_u)}$ .
  2. Solving for optimal dynamic policy path with commitment.

## Gains From Commitment Under a Simple Rule

$$x_t^c = -\omega u_t$$

- Plug into Phillips:

$$\begin{aligned}\pi_t^c &= E_t \left\{ \sum_{s=0}^{\infty} \beta^s (\kappa x_{t+s}^c + u_{t+s}) \right\} \\ &= (1 - \omega\kappa) E_t \left\{ \sum_{s=0}^{\infty} (\beta\rho_u)^s u_t \right\} \\ &= \frac{1 - \omega\kappa}{1 - \beta\rho_u} u_t\end{aligned}$$

- To interpret, note that

$$\pi_t^c = \frac{\kappa}{1 - \beta\rho_u} x_t^c + \frac{u_t}{1 - \beta\rho_u}$$

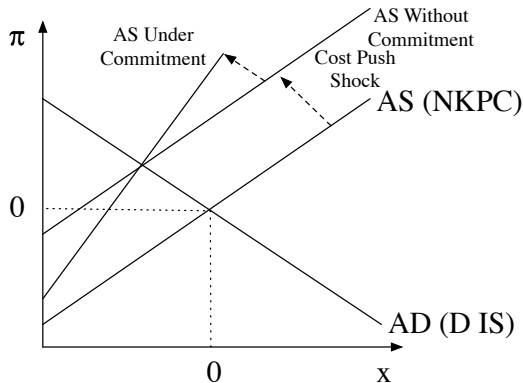
## Gains From Commitment Under a Simple Rule

$$\pi_t^c = \frac{\kappa}{1 - \beta\rho_u} x_t^c + \frac{u_t}{1 - \beta\rho_u}$$

- When push output gap down 1%, inflation falls by  $\frac{\kappa}{1 - \beta\rho_u} \%$ .
  - $\frac{\kappa}{1 - \beta\rho_u} > \kappa$ , the elasticity under discretion.
  - Thus the rule *improves the output-inflation tradeoff* for a central bank trying to stabilize inflation in response to cost push shocks.
- Extra kick due to impact of policy rule on expectations in *future course of output gap*.
  - Expected to remain high tomorrow if  $\rho_u > 0$ .
  - Respond aggressively tomorrow, which improves tradeoff today.
  - Central bank that commits to high  $\omega$  is able to signal that it will sustain an aggressive response to persistent supply shock

## Gains From Commitment: Intuition

- Commitment twists AS curve so it is steeper
  - At lower  $x_t$ , committing to  $\pi_t$  down more in future which brings down intercept.
  - Steeper is more advantageous to central bank because smaller  $x_t$  decline to stabilize  $\pi_t$  in the face of cost-push shock.



## Gains From Commitment Under a Simple Rule

- What is the optimal  $\omega$ ?

$$\min_{\omega} \frac{1}{2} E_t \left\{ \sum_{s=0}^{\infty} \beta^s \left[ \vartheta (-\omega u_{t+s})^2 + \left( \frac{1 - \omega \kappa}{1 - \beta \rho_u} u_{t+s} \right)^2 \right] \right\}$$

$$FOC : x_t^c = -\frac{\kappa}{\vartheta (1 - \beta \rho_u)} \pi_t^c$$

- The optimal commitment rule can be obtained under discretion by appointing a central banker with a weight on output of:

$$\vartheta^c = \vartheta (1 - \beta \rho_u) < \vartheta$$

- A conservative central banker relative to societal preferences!
  - This is how society can “tie itself to the mast.”
- Intuition: Reputation for aggressive and persistent response in the future helps tradeoff today.
- Trading off gains from commitment against responding more aggressively to inflation than society prefers, so do not pick someone too hawkish.

## Generalized Optimal Policy With Commitment

- We now turn to the general case:

$$\min_{\{x_{t+s}, \pi_{t+s}\}_{s=0}^{\infty}} \frac{1}{2} E_t \left\{ \sum_{s=0}^{\infty} \beta^s [\vartheta x_{t+s}^2 + \pi_{t+s}^2] \right\}$$

subject to

$$\hat{\pi}_t = \kappa x_t + \beta E_t \{\hat{\pi}_{t+1}\} + u_t$$

- Letting  $\xi_t$  be Lagrange multipliers on the period  $t$  Phillips and *t.i.p.* be terms independent of policy. The Lagrangian is then:

$$\mathcal{L} = \frac{1}{2} E_t \left\{ \sum_{s=0}^{\infty} \beta^s \left[ \begin{array}{c} (\pi_{t+s}^2 + \vartheta x_{t+s}^2) \\ + 2\xi_{t+s} (\pi_{t+s} - \kappa x_{t+s} - \beta \pi_{t+s+1}) \\ + t.i.p. \end{array} \right] \right\}$$



## Generalized Optimal Policy With Commitment

$$\mathcal{L} = \frac{1}{2} E_t \left\{ \sum_{s=0}^{\infty} \beta^s \left[ \begin{aligned} &(\pi_{t+s}^2 + \vartheta x_{t+s}^2) \\ &+ 2\xi_{t+s} (\pi_{t+s} - \kappa x_{t+s} - \beta \pi_{t+s+1}) + t.i.p. \end{aligned} \right] \right\}$$

- Setting  $\beta = 1$  to simplify algebra, the FOC are:

$$\begin{aligned} \vartheta x_{t+s} - \kappa \xi_{t+s} &= 0 \\ \pi_{t+s} + \xi_{t+s} - \xi_{t+s-1} &= 0 \quad \forall s \geq 1 \\ \pi_t + \xi_t &= 0 \end{aligned}$$

- Combining, we have:

$$\begin{aligned} x_t &= -\frac{\kappa}{\vartheta} \pi_t \\ \Delta x_{t+s} &= -\frac{\kappa}{\vartheta} \pi_{t+s} \quad \forall s \geq 1 \end{aligned}$$

- This is the same as under discretion, but with a first *difference rule*.

# Generalized Optimal Policy With Commitment

$$\begin{aligned}x_t &= -\frac{\kappa}{\vartheta}\pi_t \\ \Delta x_{t+s} &= -\frac{\kappa}{\vartheta}\pi_{t+s} \quad \forall s \geq 1\end{aligned}$$

- These can be combined into:

$$x_{t+s} = -\frac{\kappa}{\vartheta}\hat{p}_{t+s} \quad \forall s \geq 0$$

where  $\hat{p}_{t+s} = p_{t+s} - p_{t-1}$  is the log deviation between the current price level and an “implicit target” given by the price level one period before the central bank commits to the policy.

- This is a “flexible” *price level targeting* rule.

## Generalized Optimal Policy With Commitment: Intuition

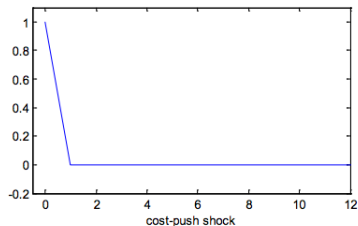
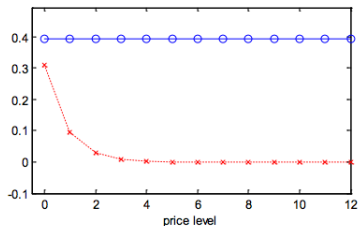
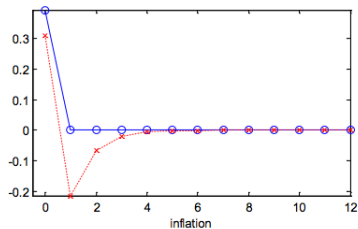
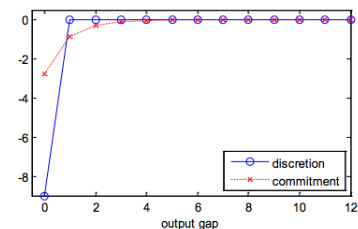
$$x_{t+s} = -\frac{\kappa}{\vartheta} \hat{p}_{t+s} \quad \forall s \geq 0$$

- Not time consistent.

$$\begin{aligned} x_t &= -\frac{\kappa}{\vartheta} \pi_t \\ \Delta x_{t+s} &= -\frac{\kappa}{\vartheta} \pi_{t+s} \quad \forall s \geq 1 \end{aligned}$$

- Each period, want to let today be the base period, choose discretionary policy that period, and then follow rule in future.
- Also, may violate Taylor Principle (see Gali).
- To understand intuition, look at impulse responses to temporary and persistent cost-push shocks.

# Intuition: Impulse Response to Temporary Cost-Push Shock



## Intuition: Impulse Response to Temporary Cost-Push Shock

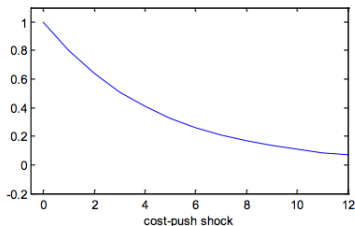
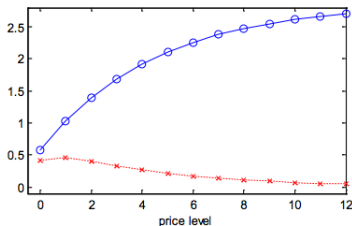
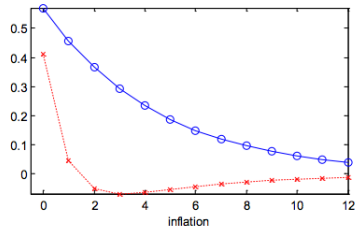
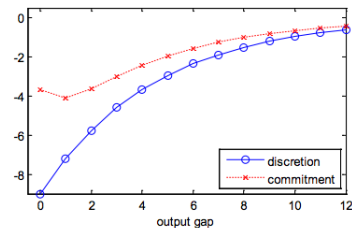
- Under discretion, reduce  $x_t$  in response to cost push shock, offsetting some of its effect.
  - This does not persist once shock has vanished.
- Under commitment, continue to reduce  $x_{t+s}$  as long as  $p_{t+s}$  is above target, well beyond when shock has vanished.
- Why does central bank maintain persistently negative output gap even after shock vanishes?
  - The credible threat to continue to contract  $x_t$  in the future dampens current inflation.
  - This reduces the effect of cost push shock and improves tradeoff between output gap and inflation this period.
  - Can see this from Phillips iterated forward:

$$\pi_t = \kappa x_t + \kappa \sum_{s=1}^{\infty} \beta^s E_t \{x_{t+s}\} + \frac{1}{1 - \beta \rho_u} u_t$$

# Intuition: Impulse Response to Temporary Cost-Push Shock

- Generally, discretion involves more stabilization in medium term than commitment policy calls for.
  - Because do not internalize benefits in terms of short-term stability that result from allowing larger deviations of output gap in the future.
- Discretionary policy thus has a *stabilization bias*.

# Intuition: Impulse Response to Persistent Cost-Push Shock



# Monetary Policy Rules

- We can now see why many monetary economists advocate rules-based policy.
  - Things improve if “tie self to mast,” so do it!
- But should we adopt a rules-based policy in practice? And if so, which one?



# Monetary Policy Rules: Inflation Targeting

- *Inflation targeting* is the most popular policy.
  - New Zealand pioneered it in 1990, Chile in 1991, many countries since.
  - Fed adopts explicit 2% target in 2012, although coupled with a dual mandate that includes maximum employment.
- Why inflation targeting?
  - “Simple” rule that is easy to understand.
  - Guarantee to avoid pre-Volcker mistakes.
  - Create credibility for central bank by committing to higher effective weight on inflation in objective function.
- Generally “flexible” targeting.

# Monetary Policy Rules: Price Level Targeting

- What about price level targeting?
  - It is the general optimum in case with commitment.
- Problems:
  1. If overshoots, need to pursue deflation. In general, deflation is considered bad.
    - Under inflation targeting, let “bygones be bygones” and avoid this.
  2. Price level is hard to measure. Don't want measurement error pushing policy.
  3. Net reduction in price uncertainty is small relative to inflation targeting.
- For these reasons, many advocate inflation targeting instead of price level targeting.

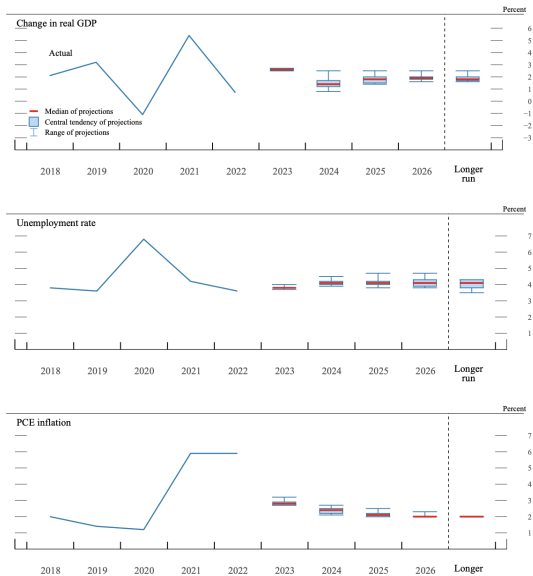
# Monetary Policy Rules: Nominal GDP Targeting

- A final option is nominal GDP targeting.
  - Idea is that under uncertainty, adjust to linear combination of expected inflation, output gap, and demand shock.
  - Nominal GDP targeting achieves this in simple way.
  - In 2012, Mike Woodford makes splash advocating this at ZLB (our next topic).
- Some Issues:
  1. What happens if there is shift in trend growth of real GDP?
  2. Policy may be overly restrictive, as places equal weights on  $x_t$  and  $p_t$  when optimal policy does not call for it.
  3. Calls for Fed to tighten when nominal GDP growth is high in recovery from recession.

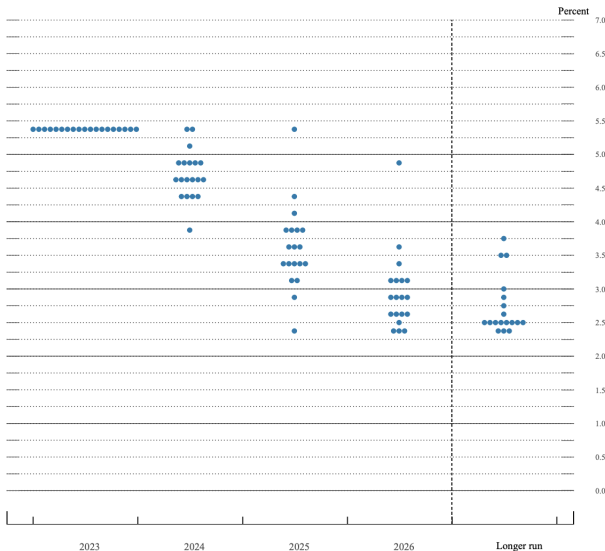
# Communication and Central Bank Credibility

- Many Central Banks maintain discretion.
  - Maintains flexibility, particularly in crises.
  - Allows Fed to weight incoming data itself, rather than relying on a rule.
- However, key takeaway from rules literature is that Central Bank can benefit from credibility.
  - Leads to increased emphasis on communication and credibility.
  - Credibility is the most crucial thing in monetary policy  $\Rightarrow$  tendency to follow through on commitments conditional on data not changing.
- Examples:
  - Focus on monitoring inflation and responding aggressively in policy statement.
  - Quarterly press conferences by Fed Chair and releases of FOMC participant forecasts.

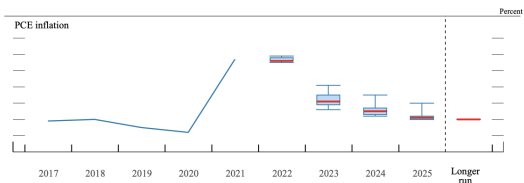
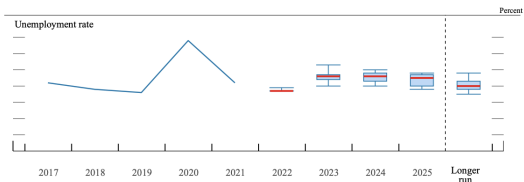
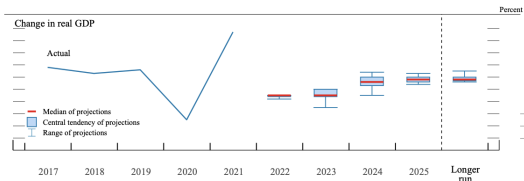
# December 2023 Forecasts of Fundamentals



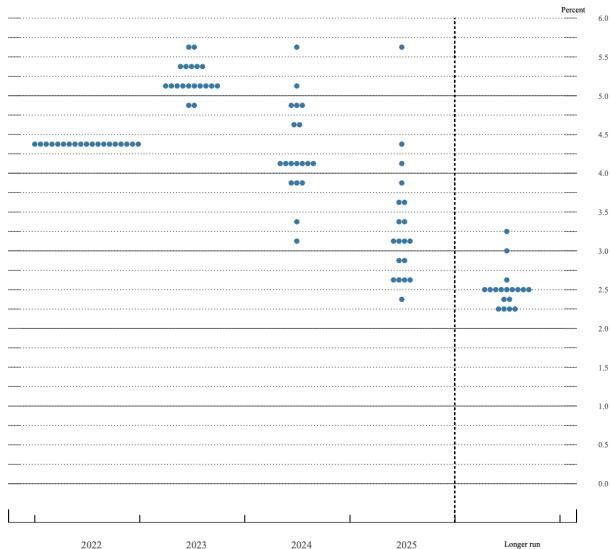
# December 2023 Interest Rate Forecasts



# December 2022 Forecasts of Fundamentals

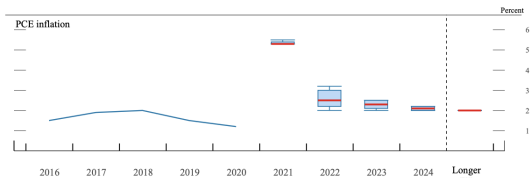
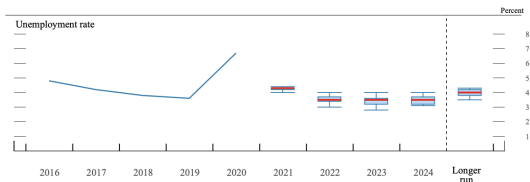
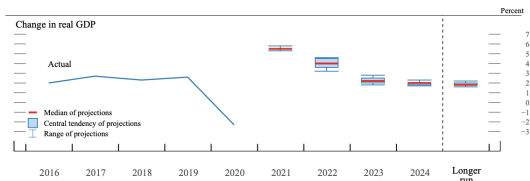


# December 2022 Interest Rate Forecasts



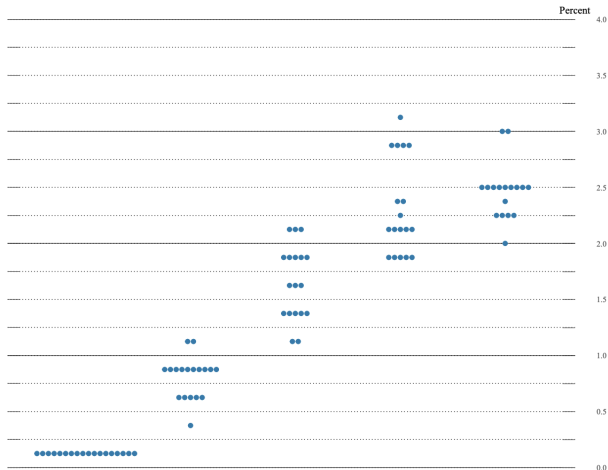


# December 2021 Forecasts of Fundamentals

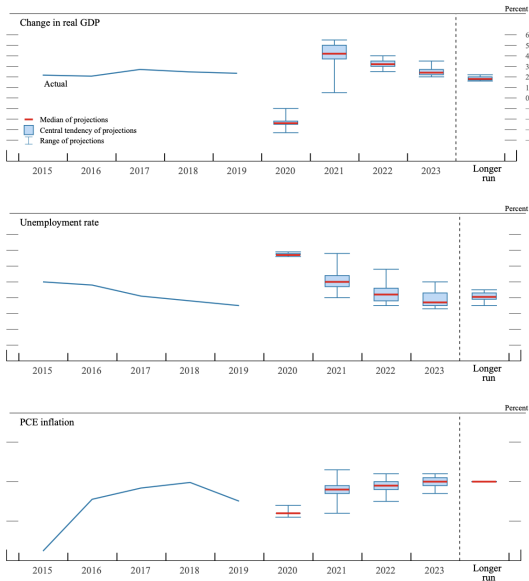


# December 2021 Interest Rate Forecasts

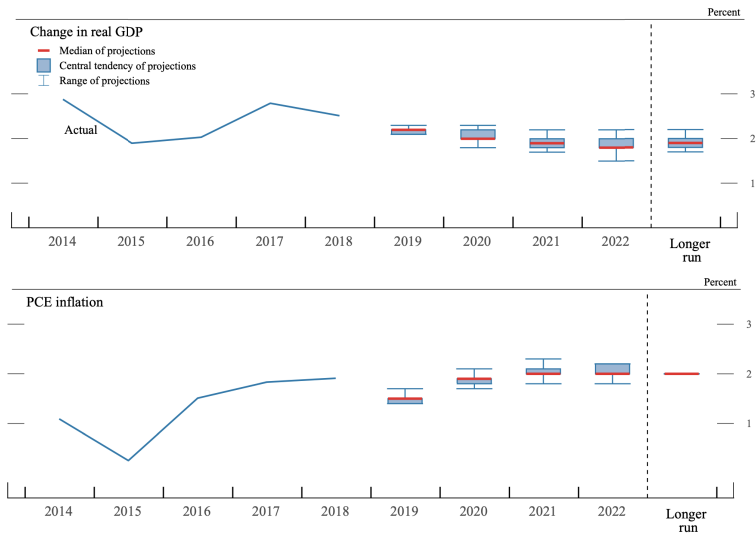
Figure 2. FOMC participants' assessments of appropriate monetary policy: Midpoint of target range or target level for the federal funds rate



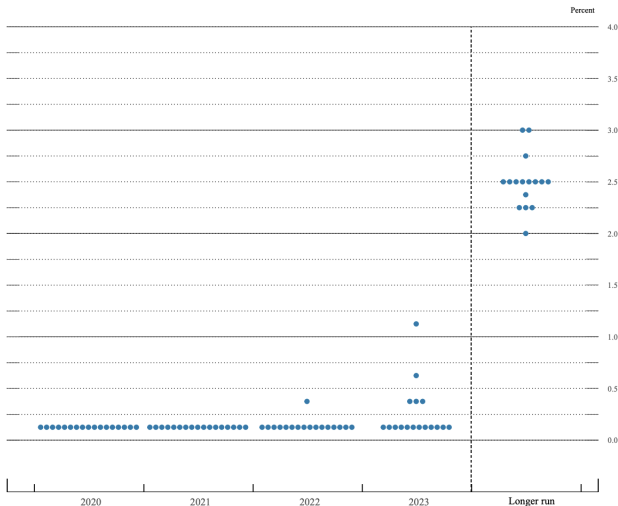
# December 2020 Forecasts of Fundamentals



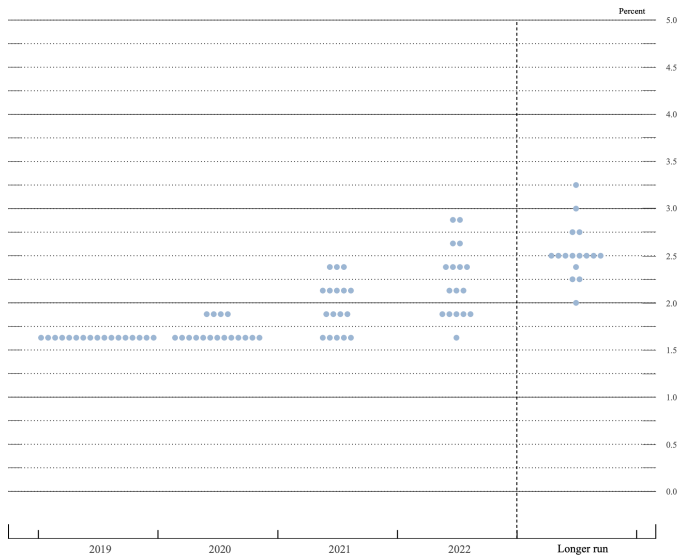
# December 2019 Forecasts of Fundamentals



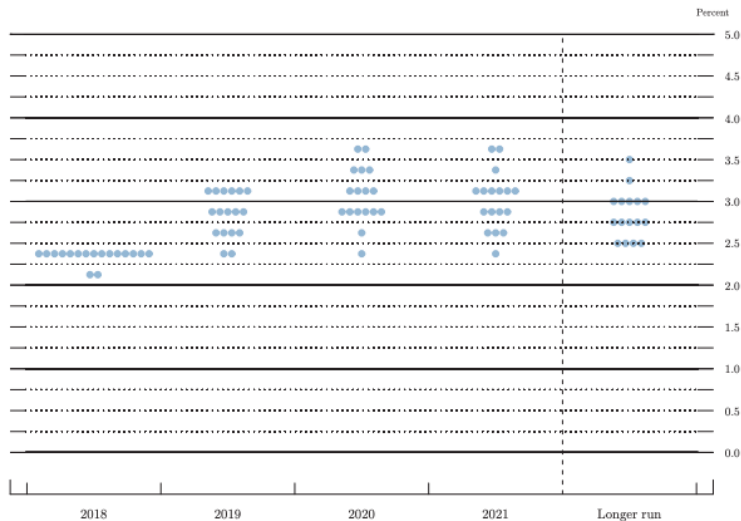
# December 2020 Interest Rate Forecasts



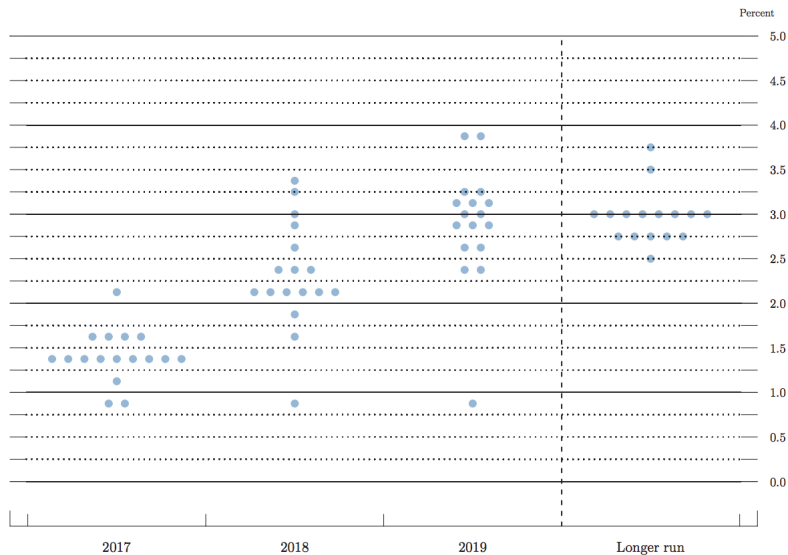
# December 2019 Interest Rate Forecasts



# December 2018 Interest Rate Forecasts



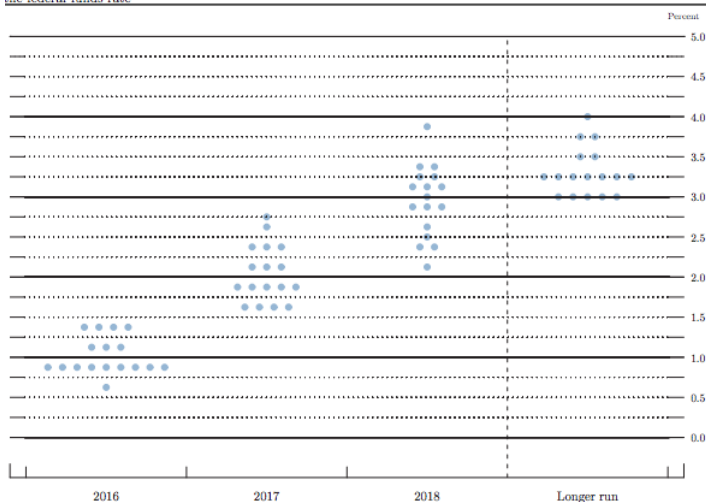
# March 2017 Interest Rate Forecasts





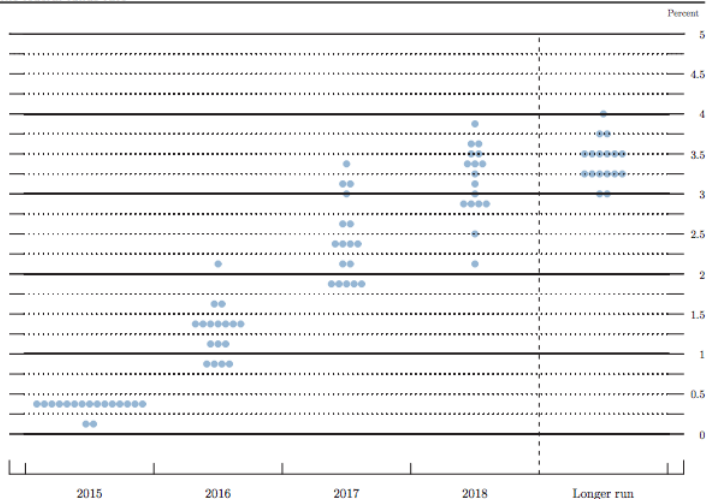
# March 2016 Interest Rate Forecasts

Figure 2. FOMC participants' assessments of appropriate monetary policy: Midpoint of target range or target level for the federal funds rate



# December 2015 Interest Rate Forecasts

Figure 2. FOMC participants' assessments of appropriate monetary policy: Midpoint of target range or target level for the federal funds rate



# Whither Fiscal Policy?

- While “Science of Monetary Policy” (according to Clarida, Gali, and Gertler) emerged, fiscal policy falls into political morass.
- Perhaps not less effective, but certainly less nimble.
- Consensus: Fiscal policy takes back seat for stabilization (and less work on it, until recently).

## Consensus in Macro?

- In 2008, Blanchard writes paper on “The State of Macro” concluding that the “battlefield” of the 1970s has given way to a “largely shared vision both of fluctuations and methodology.” He concludes “the state of macro is good.”
- Monetary policy by principles and rules we have discussed.
  - Generally thought to be able to stabilize fairly well
    - “Great Moderation”
  - Thought to be more nimble and independent than fiscal policy.
- And then the Great Recession happened...

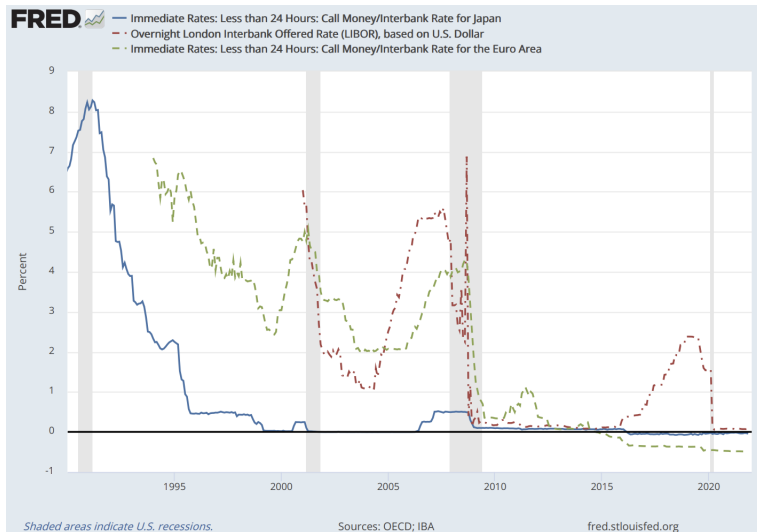
# The Intuition of Sticky Prices and Monetary Policy

- In theory, prices should adjust a lot, quantities relatively little.
- Sticky prices: If limit price movement, quantities adjust more.
- Example: Surprise money expansion.
  - Wages and prices should all double, with no effect.
  - If prices and wages are sticky, output rises in short run.
- However, always one price that *can* adjust.
  - Interest rate: Price of consumption today vs. tomorrow.
  - Interest rates act as a stabilizer, making sure sticky prices do not do “too much” because this key price is flexible.
- This is how monetary policy stabilizes economy:
  - Moving  $i_t$  adjusts  $r_{t+1}$  relative to  $r_{t+1}^n$ , which through intertemporal substitution along Euler equation expands or contracts aggregate demand.
  - Demand side instrument: no tradeoff for demand shocks, only for supply shocks.

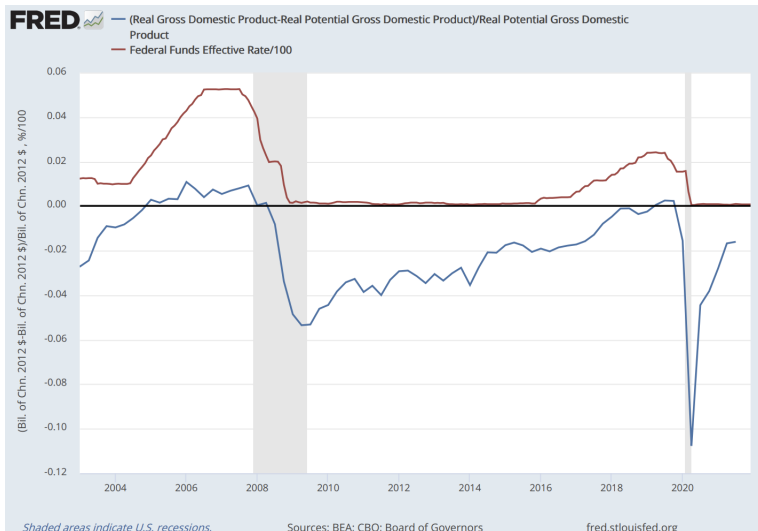
# The Liquidity Trap

- But what if interest rates are *also* stuck?
  - Then quantities will adjust a *lot* because this key intertemporal price fails to fully adjust.
- We call this situation a *liquidity trap*.
  - Topsy-turvy world in which most conventional intuition is flipped on its head.
- How could a liquidity trap occur?
  - Central bank hits *zero lower bound* on nominal interest rates.
  - Below  $i = 0$ , money demand blows up. No demand for bonds. Open market operations useless.
  - Because  $r_{t+1} = i_t - E_t \{\pi_{t+1}\}$ , happens when full employment real interest rate falls below  $-E_t \{\pi_{t+1}\}$ .
- Keynes described liquidity trap, but until late 1990s, seen as a theoretical curiosity.

# The World in a Liquidity Trap

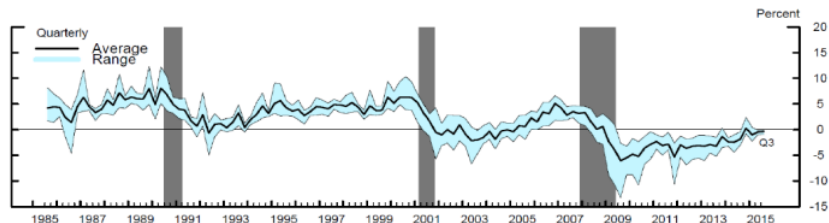


# Liquidity Trap and Recession





# Model Implied Natural Rate of Interest



Note: The shaded bars indicate periods of business recession as defined by the National Bureau of Economic Research.

Source: The estimates are drawn from four models: (1) a dynamic stochastic general equilibrium (DSGE) model developed by the staff of the Federal Reserve Board and described in Kiley (2013); (2) a DSGE model developed by the staff of the Federal Reserve Bank of New York and described in Del Negro and others (2013); (3) a DSGE model developed by the staff of the Federal Reserve Board based on Christiano, Motto, and Rostagno (2014); and (4) a DSGE model developed by the staff of the Federal Reserve Board based on Guerrieri and Iacoviello (2013 [rev. 2014]).

## This Class' Approach to the Liquidity Trap

- Since Great Recession, burgeoning literature.
  - Too much to cover, very technical.
  - Ignore complications: multiple equilibria, non-linearities.
- I will try to give you broad outlines of what NK model tells us about a liquidity trap, focusing on policy.
- Will primarily use a simple NK model with a deterministic liquidity trap (as in Werning, 2012).
  - *Exogenous* liquidity trap where the real rate is negative.
  - Questions: Why is a liquidity trap so destructive? What is best set of policies given liquidity trap?
- Will not cover where liquidity trap comes from.
  - Key idea: Deleveraging by indebted can force savings enough to drive interest rate determined by saver Euler negative.
  - See Eggertsson and Krugman (2012) for simple treatment of endogenous liquidity trap, Korinek and Simsek (2016) for application to macroprudential policy.

## Outline: Questions on the Liquidity Trap

1. What Is the Effect of a Liquidity Trap in the NK Model?
2. What Is Optimal Monetary Policy in a Liquidity Trap?
  - 2.1 Forward Guidance (Gali 5.4)
  - 2.2 Other Unconventional Policies
  - 2.3 Is Zero the Lower Bound?
3. What Is the Role of Fiscal Policy in a Liquidity Trap?

# What Is the Effect of a Liquidity Trap in the NK Model?

- Start with standard NK model with no cost-push shocks:

$$\pi_t = \beta E_t \{ \pi_{t+1} \} + \kappa x_t$$

$$x_t = E_t \{ x_{t+1} \} - \sigma E_t \{ i_t - \pi_{t+1} - r_{t+1}^n \}$$

- Optimal monetary policy is to set  $i_t = r_{t+1}^n$  so  $x_t = 0$  and  $\pi_t = 0$  (divine coincidence).
- Thought experiment we will use repeatedly today:
  - The natural rate is at its steady state of  $\rho$  until period  $t - 1$ .
  - At period  $t$ , learn  $r_{t+1}^n$  will follow deterministic path:

$$r_{t+1}^n = \begin{cases} -\Delta < 0 & \text{from } t \text{ to } t + T \\ \rho & \text{from } t + T + 1 \text{ on} \end{cases}$$

- For now, Central Bank pursues optimal discretionary policy
  - Prior to  $t$  and from  $t + T + 1$  onwards,  
set  $x_t = -\frac{\kappa}{\vartheta} \pi_t \Rightarrow i_t = \rho \Rightarrow \pi_t = 0$ .
  - From  $t$  to  $t + T$ , lower  $i_t$  to ZLB so  $i_t = 0$ .

# What Is the Effect of a Liquidity Trap in the NK Model?

- Iterating forward we have:

$$x_t = -\sigma E_t \left\{ \sum_{s=0}^{\infty} \left[ \left( \hat{i}_{t+s} - \hat{\pi}_{t+s+1} - \hat{r}_{t+s+1}^n \right) \right] \right\}$$

$$\pi_t = E_t \left\{ \sum_{s=0}^{\infty} \beta^s \kappa x_{t+s} \right\}$$

- Deterministic path so can drop expectations. Split into two sums, one from 0 to  $T$  and one from  $T+1$  to  $\infty$ :

$$x_t = -\sigma \sum_{s=0}^T \left( \hat{i}_{t+s} - \hat{\pi}_{t+s+1} - \hat{r}_{t+s+1}^n \right) - \underbrace{\sigma \sum_{s=T+1}^{\infty} \left( \hat{i}_{t+s} - \hat{\pi}_{t+s+1} - \hat{r}_{t+s+1}^n \right)}_{\text{Zero By Divine Coincidence}}$$

$$\pi_t = \sum_{s=0}^T \beta^s \kappa x_{t+s} + \underbrace{\sum_{s=T+1}^{\infty} \beta^s \kappa x_{t+s}}_{\text{Zero By Divine Coincidence}}$$

## What Is the Effect of a Liquidity Trap in the NK Model?

- Plugging in optimal policy in liquidity trap of  $i_t = 0$  and  $r_{t+1}^n = -\Delta$ , we have:

$$x_t = -\sigma \sum_{s=0}^T (\Delta - \pi_{t+s+1})$$

$$\pi_t = \sum_{s=0}^T \beta^s \kappa x_{t+s}$$

- This implies persistent slump with  $x_t < 0$  and  $\pi_t < 0$ !
  - Start in period  $t + T$ . Know  $\pi_{t+T+1} = 0$  and  $\Delta > 0$ , so  $x_{t+T} < 0$  and  $\pi_{t+T} < 0$ .
  - In period  $t + T - 1$ ,  $\pi_{t+T} < 0$  and  $\Delta > 0$ , so  $x_{t+T-1} < x_{t+T} < 0$  and  $\pi_{t+T-1} < \pi_{t+T} < 0$ .
  - Working backward,  $x < 0$  and  $\pi < 0$  all the way back to period  $t$ , with bigger output gaps and deflation farther back.

## What Is the Effect of a Liquidity Trap in the NK Model?

- Why the big slump?
- Even if inflation were zero, consumption would be depressed by

$$x_t = -\sigma \sum_{s=0}^T \Delta$$

- Households are saving “too much” because  $r_t$  is “too high.”
- Key Idea: Deflation exacerbates the ZLB.
  - Deflation occurs because negative output gaps push down MC.
  - This pushes  $r_t$  higher as  $r_t = -E_t \{\pi_{t+1}\}$ , which makes  $x_t$  lower, leading to more deflation....

$$x_t = -\sigma \sum_{s=0}^T (\Delta - \pi_{t+s+1})$$

- Inflation is forward looking, so deflation is worst at the beginning and then gets better.
- Next: Policy at ZLB. Read Gali 5.4